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QUANTUM STORAGE AND CLONING OF LIGHT STATES IN EIT-LIKE MEDIUM

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In the paper we consider a new approach for storage and cloning of quantum information by three level atomic (molecular) systems in the presence of the electromagnetically induced transparency (EIT) effect. For that, the various schemes of transformation into the bright and dark polaritons for quantum states of optical field in the medium are proposed. Physical conditions of realization of quantum nondemolition (QND) storage of quantum optical state are formulated for the first time. We have shown that the best storage and cloning of can be achieved with the atomic ensemble in the Bose-Einstein condensation state. We discuss stimulated Raman two-color photoassociation for experimental realization of the schemes under consideration.

Keywords: EIT; information.

1. Introduction

Modern development of quantum information, communication and especially quantum cryptography requires the new methods and approaches in quantum information processing. A significant progress on this way can be achieved with the help of novel devices for quantum storage, memory and transmission of information embodied in optical continuous variables, i.e. in Hermitian quadratures¹. Typically, the memory devices proposed now for those purposes explore various methods of entanglement and mapping of quantum state of light onto the atomic ensembles²⁻⁴.

One practically important and interesting possibility to store optical information is connected with the usage of effect of electromagnetically induced transparency (EIT) taking place in three level atomic systems with Λ -configuration of the energy levels - see Fig.1. In this case strong (classical) coupling field (i.e. control field) cre-

ates transparency window in the medium, and second weak (quantum) probe field (we call it as a signal) propagates through the resonant atomic system with a very small absorption^{3–5}. The EIT effect is accompanied with significant reduction of observable group velocity of signal pulse, as well. In Ref. 4 the authors propose to use such a “stopped” light for coherent (classical) storage of information in ultracold ($T \simeq 450$ nK) sodium atoms near the transition to the Bose-Einstein condensation (BEC) state. In fact, the phenomenon is displayed by switching of signal pulse with delay time $\tau \simeq 45$ μ sec and more. The Doppler effect, being limitation for the process, can be suppressed for the case. Physically, quantum properties of the coupling system, i.e. signal optical pulse and atomic system under the EIT condition, can be described in terms of “bright” and “dark” polaritons for atom-field excitations - see Refs. 3, 5 for more details. The experiment has been carried out in Ref. 6 for hot ($T \simeq 360$ K) atomic Rb vapor cell, and quantum state transfers from the light to atomic excitations.

Another approach to store and transmit quantum information by memory devices has been established in Ref. 1. In particular, the fidelity up to 70% and memory lifetime of up to 4 msec has been achieved experimentally for Gaussian light pulses using complex (three step passing) scheme for light-atom interaction and subsequent measurements by feedback introduced onto the atoms.

In present paper we consider also the problem of storage of quantum state of optical field by using the quantum cloning procedure for continuous variables for the light-atom interaction⁷. It is shown^{8–10} that although perfect cloning is forbidden in quantum theory the fidelity of such a process can be high enough and equal 2/3.

In Section 2 we propose quantum non-demolition (QND) method to store the quantum state of light what is very closed to QND measurement procedure in quantum and atomic optics^{11, 12}. In this case the specific properties of quantum state of storage device become important. Necessary criteria are formulated for estimation of efficiency of optical field storage and transmission. In Section 3 we consider a new scheme for optimal quantum cloning of optical field in the EIT-like system. In the case a preliminary linear amplification of the signal optical pulse is absolutely necessary. In conclusion we briefly discuss appropriate application of the schemes proposed in the paper for the problem of quantum cryptography with continuous variables. In Appendix we also discuss an implementation of the stimulated two-photon Raman photoassociation^{13, 14} effect under the BEC condition as a tool for experimental realization of the problem under consideration.

2. Quantum non-demolition storage of the state of optical field

Let us consider interaction of three level atomic system with two optical fields under the three effectively interacting quantum modes approximation. In this regime all atoms macroscopically occupy three internal levels depicted in Fig.1. The three-mode approach is valid when the spatial state (motional degree of freedom) of atomic cloud still unchanged on the time of atom-field interaction and also still essentially

independent on internal atomic states. For ultracold atoms near the condensation transition such a conditions are fulfilled for small number of particles ($N \lesssim 10^4$) or relevant size of medium and trap size (spacing between the trap modes $\hbar\omega_{trap}$ that is numerically corresponds to frequency $\omega_{trap} \leq 100\text{Hz}$)¹⁵. The macroscopic atomic coherence can be also achieved for hot atomic clouds. There exist some theoretical and experimental proposals for that – see e.g. Ref. 16. In particular, the atomic system that is described by collective magnetic momentum J of ground state is especially have been prepared in coherent spin state for which only one projection, say J_x , have non-zero average value for current experiments^{1,2}. Another possibility, what is important for us in the paper, is based on achieving of EIT regime for atom-field interaction⁶.

Thus, we will describe the bright (light) and dark (matter) polaritons in EIT-like medium (being the result of excitation of quantum coupled state for optical field and matter) by unitary linear transformation for two annihilation operators Φ_f and Φ_ξ , respectively, in the form:

$$\Phi_f = \mu f_{in} - \nu \xi_{in}, \quad \Phi_\xi = \mu \xi_{in} + \nu f_{in}, \quad (1)$$

where f_{in} is the annihilation operator for signal (probe) field at the input of atomic medium. In the Schwinger bosons representation¹⁷ spin operator $\xi = a_2^+ a_1 / \sqrt{N}$ characterizes macroscopic excitations in atomic system, a_j (a_j^+) is the atom annihilation (creation) operator for j -th internal level ($j = 1, 2$) respectively, N is the total number of particles. The parameters $\mu = \cos(\theta)$ and $\nu = \sin(\theta)$ in Eq. (1) depend on oblique angle θ between signal and control field. The value θ defines the efficiency of linear transformation (1) and energy exchange for polaritons. In the presence of the EIT effect it can be represented as (see Ref. 3):

$$\frac{\nu^2}{\mu^2} = \tan^2(\theta) = \frac{\Omega_2^2 N}{\Omega_1^2} = n_g, \quad (2)$$

where Ω_j ($j = 1, 2$) characterizes the Rabi frequency for control and signal fields respectively, n_g is the refractive index.

In adiabatic approximation we assume that the atoms are populated mostly at the lowest (ground) level $|2\rangle$, and following inequalities are valid for average number of atoms $n_j = \langle a_j^+ a_j \rangle$ ($j = 1, 2, 3$) - see also Fig.1:

$$n_3 \ll n_1 < n_2. \quad (3)$$

With the help of expressions (1) for quadrature components of polaritons $Q_j^{out} = \Phi_j + \Phi_j^+$, $P_j^{out} = i(\Phi_j^+ - \Phi_j)$, $j = f, \xi$ one can obtain for output of the system:

$$\begin{aligned} Q_f^{out} &= \mu Q_f^{in} - \nu Q_\xi^{in}, & P_f^{out} &= \mu P_f^{in} - \nu P_\xi^{in}, \\ Q_\xi^{out} &= \mu Q_\xi^{in} + \nu Q_f^{in}, & P_\xi^{out} &= \mu P_\xi^{in} + \nu P_f^{in}, \end{aligned} \quad (4)$$

where quadratures $Q_f^{in} = f_{in} + f_{in}^+$, $P_f^{in} = i(f_{in}^+ - f_{in})$ relate to signal field at the input of the medium. The values $Q_\xi^{in} = \xi_{in} + \xi_{in}^+ \equiv (a_1^+ a_2 + a_2^+ a_1) / \sqrt{N}$ and

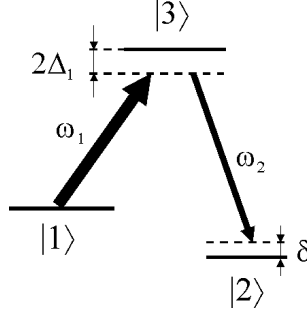


Fig. 1. Energy levels for Λ -scheme under the EIT effect; ω_1 (ω_2) is the frequency of control (signal) field, $2\Delta_1$ and δ are the detunings from the exact resonance.

$P_\xi^{in} = i(\xi_{in}^+ - \xi_{in}) \equiv i(a_1^+ a_2 - a_2^+ a_1) / \sqrt{N}$ describe the elementary excitations (i.e. the spin wave components) and satisfy the standard SU(2)-algebra commutation relations.

We consider, further, the quantum non-demolition (QND) storage of information by signal field quadratures Q_f^{in} and P_f^{in} measurement. The procedure is shown in Fig.2: the measurement of polariton quadratures Q_ξ^{out} and P_ξ^{out} at the output of the device gives information on the values of signal quadratures Q_f^{in} and P_f^{in} at the input.

Let us introduce following correlation coefficients to describe the process under consideration in Heisenberg representation (cf. Ref. 12):

$$C_1^{(X)} = \frac{|\langle X_f^{in} X_f^{out} \rangle - \langle X_f^{in} \rangle \langle X_f^{out} \rangle|^2}{V_{X,f}^{in} V_{X,f}^{out}}, \quad (5)$$

$$C_2^{(X)} = \frac{|\langle X_f^{in} X_\xi^{out} \rangle - \langle X_f^{in} \rangle \langle X_\xi^{out} \rangle|^2}{V_{X,f}^{in} V_{X,\xi}^{out}}, \quad (6)$$

$$C_3^{(X)} = \frac{|\langle X_f^{out} X_\xi^{out} \rangle - \langle X_f^{out} \rangle \langle X_\xi^{out} \rangle|^2}{V_{X,f}^{out} V_{X,\xi}^{out}}, \quad (7)$$

where continuous variable $X = \{Q, P\}$; $V_{X,f}^{in(out)} = \langle (\Delta X_f^{in(out)})^2 \rangle$, $V_{X,\xi}^{in(out)} = \langle (\Delta X_\xi^{in(out)})^2 \rangle$ are the variances of quadrature components for signal pulse and polariton, respectively, before (after) storage procedure.

The conditions (5)–(7) take into account the correlations for two quadratures Q_f^{in} and P_f^{in} simultaneously (in contrast with the quantum non-demolition measurement procedure for a single quadrature component Q_f^{in} or P_f^{in} (see e.g. Ref. 11), and have a simple physical meaning. The correlation coefficient $C_1^{(Q,P)}$ takes into account the

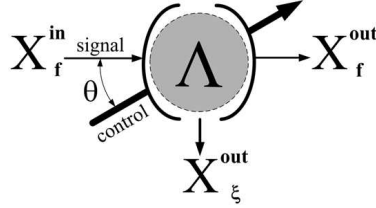


Fig. 2. Scheme of the QND storage of the X_f^{in} -quadratures ($X = \{Q, P\}$) for signal field in the EIT-like medium (Λ); θ is the oblique angle between control and signal fields. The $X_{f,\xi}^{out}$ are the polariton quadratures at the output of the medium.

degree of distortion for quadratures Q_f^{in} and P_f^{in} of input signal at the output of the read-out device. The “quality” of non-demolition recording of optical information for that can be established by using the $C_2^{(Q,P)}$ coefficients. The ability of the storage device to prepare the quantum state for signal field in respect of further transmission is characterized by the $C_3^{(Q,P)}$ coefficients. In other words, the coefficients $C_3^{(Q,P)}$ can be associated with the cloning procedure for signal pulse into the considered atomic ensemble. In fact, the coefficients in Eq. (7) represent a necessary condition for such a procedure - see also Eq. (19) below.

In the case of the ideal QND-storage the all correlation coefficients are equal to one, i.e. $C_j^{(X)} = 1$, $X = Q, P$, $j = 1, 2, 3$. The fact occurs when

$$\begin{aligned} Q_f^{out} &= Q_f^{in}, & Q_\xi^{out} &= G Q_f^{in} \\ P_f^{out} &= P_f^{in}, & P_\xi^{out} &= G P_f^{in}, \end{aligned} \quad (8)$$

where G is QND gain. In the case of ideal cloning procedure the value $G = 1$ - cf. (19).

From expressions (8) and Eqs. (4) it is easy to see that the EIT-like medium is not ideal for quantum information storage but in some cases can be closed to that. Actually, using the Eqs. (4), we can rewrite the expressions (5)–(7) in the form:

$$\begin{aligned} C_1^{(X)} &= \frac{\mu^2 V_{X,f}^{in}}{\mu^2 V_{X,f}^{in} + \nu^2 V_{X,\xi}^{in}}, & C_2^{(X)} &= \frac{\nu^2 V_{X,f}^{in}}{\mu^2 V_{X,\xi}^{in} + \nu^2 V_{X,f}^{in}}, \\ C_3^{(X)} &= \frac{\mu^2 \nu^2 (V_{X,f}^{in} - V_{X,\xi}^{in})^2}{(\mu^2 V_{X,f}^{in} + \nu^2 V_{X,\xi}^{in})(\mu^2 V_{X,\xi}^{in} + \nu^2 V_{X,f}^{in})}, \end{aligned} \quad (9)$$

where $X = Q, P$.

From expressions (9) follow that correlation coefficients $C_j^{(X)}$ can satisfy simultaneously to condition $C_j^{(X)} = 1$ ($j = 1, 2, 3$) only for the case when all variances $V_{Q,\xi}^{in} = V_{P,\xi}^{in} = 0$. The condition corresponds to initially prepared perfect spin-squeezed state for atomic medium^{2,19}. If it is not the case, the quality of the storage

and transmission of information depends on quantum properties of atomic system, and also is determined by oblique angle θ .

In particular, the coefficient $C_1^{(X)} \simeq 1$ for $|\nu| \ll 1$ ($\theta \ll 1$) in the limit of the non-demolition storage of signal field quadratures. When $C_{2,3}^{(X)} \rightarrow 1$, as well, we must require the following inequalities to be fulfilled for the variances:

$$V_{Q,\xi}^{in} \ll \frac{\nu^2}{\mu^2}, \quad (10)$$

$$V_{P,\xi}^{in} \ll \frac{\nu^2}{\mu^2}. \quad (11)$$

Note, that the conditions (10), (11) are not require in essential suppression of group velocity for signal pulse passing the atomic medium - see also (2).

Let us examine the criteria (10), (11) under the conditions of the experiments related to the EIT-effect observation and carried out recently^{4,6}. We assume the probe field is in arbitrary coherent state with the variances $V_{X,f}^{in} = 1$, for simplicity.

For two-mode Fock state $|\psi\rangle_{at} = |n_1\rangle |n_2\rangle$ of atomic system under condition (3) we readily find from expressions (9):

$$C_1^{(X)} \simeq \frac{\mu^2}{1 + 2\nu^2 n_1}, \quad C_2^{(X)} \simeq \frac{\nu^2}{1 + 2\mu^2 n_1}, \quad C_3^{(X)} \simeq \frac{4\mu^2 \nu^2 n_1^2}{1 + 4\mu^2 \nu^2 n_1^2 + 2n_1}. \quad (12)$$

The maximal values for coefficients $C_j^{(X)}$ can be achieved in the limit of $n_1 \ll 1$. The case corresponds to coherent atomic medium with the variance of polaritons $V_{X,\xi}^{in} = 1$. Then, one can obtain from Eqs. (9), (12):

$$C_{1,class}^{(X)} \simeq \mu^2, \quad C_{2,class}^{(X)} \simeq \nu^2, \quad C_{3,class}^{(X)} = 0. \quad (13)$$

The relations (13) for correlation coefficients (5)–(7) determine an ultimate case of classical recording of information by coherent atomic device when the conditions (10), (11) are violated.

For the Bose-Einstein condensate medium the initially prepared state

$$|\psi\rangle_{BEC} = \frac{1}{\sqrt{N!}} (\alpha_1 a_1^+ + \alpha_2 a_2^+)^N |0\rangle \quad (14)$$

is entangled^{18,19}. The parameters $|\alpha_j|^2 = n_j/N$ ($j = 1, 2$) determine a relative atomic population at the levels $|1\rangle$ and $|2\rangle$, respectively, and obey normalization condition $|\alpha_1|^2 + |\alpha_2|^2 = 1$. In this case we have for the variances $V_{X,\xi}^{in}$:

$$V_{Q,\xi}^{in} = 1 - 4|\alpha_1|^2 |\alpha_2|^2 \cos^2(\varphi), \quad V_{P,\xi}^{in} = 1 - 4|\alpha_1|^2 |\alpha_2|^2 \sin^2(\varphi), \quad (15)$$

where $\varphi = \varphi_2 - \varphi_1$ is relative atomic phase ($\alpha_{1,2} = |\alpha_{1,2}| e^{i\varphi_{1,2}}$).

As follows from relations (15), the variances $V_{Q,\xi}^{in}$ and $V_{P,\xi}^{in}$ are in spin-squeezed states simultaneously, and the values $V_{Q,\xi}^{in} = V_{P,\xi}^{in} = 1/2$ are for $|\alpha_1|^2 = |\alpha_2|^2 = 1/2$, $\varphi = \pi/4$. In this case the relevant coefficients (see Eqs. (9)) are maximal $C_{1,2}^{(X)} = 2/3$, $C_3^{(X)} = 1/9$, and we can achieve essentially quantum storage of information. The conditions (10) and (11) are fulfilled simultaneously for the value of angle $\theta \simeq \pi/4$.

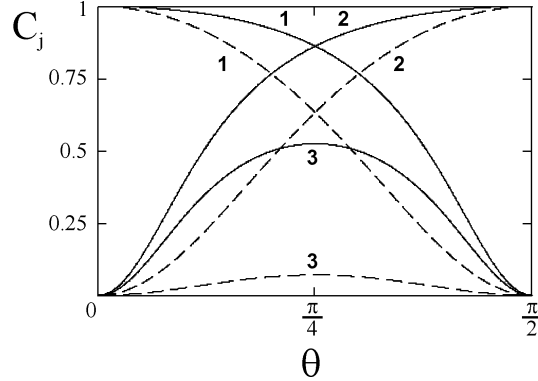


Fig. 3. Dependences of correlation coefficients C_j on the θ angle; $j = 1$ - curves 1, $j = 2$ - curves 2, and $j = 3$ - curves 3. The magnitudes of parameters used for calculations are: $|\alpha_1|^2 = 0.3$, $\varphi = \pi/4$ (dashed lines) and $\varphi = 0$ (solid lines) respectively.

The obtained dependences for correlation coefficients $C_j \equiv C_j^{(Q)}$ ($j = 1, 2, 3$) vs oblique angle θ are plotted in Fig.3.

The variances $V_{Q,\xi}^{in}$ correspond to squeezed state for the relative phase $\varphi = 0$, and the condition (10) is fulfilled only. In this regime EIT-medium performs quantum non-demolition measurement of the Q_f^{in} quadrature only. At the same time, for another quadrature component P_f^{in} the variance corresponds to coherent level of fluctuations when $V_{P,\xi}^{in} = 1$ and $C_j^{(P)} = C_{j,class}$ respectively - see Eq.(13). Practically, such a method of the quantum information storage has a narrower area of applications due to dependence on signal field quadratures Q_f^{in} and P_f^{in} .

3. Quantum cloning with polaritons

Now we switch our attention to the problem of cloning of optical field f_{in} onto the ultracold atomic (molecular) ensemble with the help of the EIT-like medium. The principal set-up for optimal cloning procedure under consideration is shown in Fig.4.

The initial radiation propagates through the phase-insensitive linear amplifier (LA) with the gain that is equal two. The linear transformation for photon annihilation operators of signal (f) and ancilla (c) modes for symmetric cloning is represented in the form:

$$f = \sqrt{2}f_{in} + c_{in}^+, \quad c_{out} = \sqrt{2}c_{in} + f_{in}^+, \quad (16)$$

where c_{in} (c_{out}) is the value of operator at the input (output) of the amplifier (we assume that c_{in} is in the vacuum state).

Then, according to the scheme in Fig.4, the signal optical field is stored in the EIT medium. The bright and dark polaritons at the output of medium represent two symmetric clones of initial field f_{in} as a result.

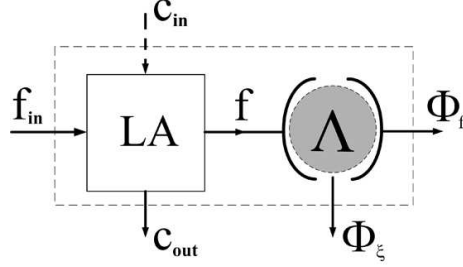


Fig. 4. Scheme of quantum cloning for light onto the polaritons. Here f_{in} (c_{in}) is annihilation operator for signal (ancilla) optical mode, respectively at the input of cloning device; LA is the linear amplifier, Λ is the EIT-like medium, Φ_f and Φ_ξ are the polariton operators at the output of apparatus.

Taking into account the Eqs. (1) and Eqs. (16) with $\theta \simeq \pi/4$ we represent a total unitary transformation for annihilation operators of bright (optical) Φ_f and dark (excitation in matter) Φ_ξ clones as:

$$\Phi_f = f_{in} + \frac{1}{\sqrt{2}} (c_{in}^+ - \xi_{in}), \quad \Phi_\xi = f_{in} + \frac{1}{\sqrt{2}} (c_{in}^+ + \xi_{in}). \quad (17)$$

Corresponding to Eqs. (17) the quadrature component transformations have the form:

$$Q_{f,\xi}^{out} = Q_f^{in} + \frac{1}{\sqrt{2}} (Q_c^{in} \mp Q_\xi^{in}), \quad P_{f,\xi}^{out} = P_f^{in} - \frac{1}{\sqrt{2}} (P_c^{in} \pm P_\xi^{in}), \quad (18)$$

where $Q_c^{in} = c_{in} + c_{in}^+$, $P_c^{in} = i(c_{in}^+ - c_{in})$ are the Hermirtian quadratures for ancilla vacuum mode at the input of amplifier (see Fig.4).

The expressions (17), (18) represent the desired linear transformations for optimal cloning procedure of continuous variables in the Heisenberg picture⁸. In the case, corresponding mean values of quadratures (18) and their variances fulfill the expressions (cf. (8)):

$$\langle X_{f,\xi}^{out} \rangle = \langle X_f^{in} \rangle, \quad (19)$$

$$V_X^{out} \equiv V_{X,f}^{out} = V_{X,\xi}^{out} = V_{X,f}^{in} + 1, \quad (20)$$

where variable $X = \{Q, P\}$.

Last term in Eq.(16b) characterizes the impossibility to perfect cloning of quantum state. For the relevant correlation coefficients (5)–(7) we have in this case:

$$C_{1,clone}^{(X)} = C_{2,clone}^{(X)} = \frac{1}{2}, \quad C_{3,clone}^{(X)} = \frac{1}{4}. \quad (21)$$

Two coefficients $C_{1,2}^{(X)}$ in relations (21) correspond to those classical values what means that optimal cloning procedure (see Exp. (18)) introduces some error to both of transmitted and stored field. At the same time the third correlation coefficient

$C_3^{(X)}$ in Exp. (21) demonstrates essentially non-classical correlation between two clones at the output of cloning device in Fig.4.

Last time the fidelity criterion is used to describe the processing of quantum information – see e.g. Refs.8–10,20. For optimal and symmetric cloning procedure of coherent optical field the fidelity F can be evaluated as:

$$F = \frac{2}{\sqrt{(2 + V_Q)(2 + V_P)}}, \quad (22)$$

where $V_Q = V_Q^{out} - V_Q^{in}$, and $V_P = V_P^{out} - V_P^{in}$ are variances that correspond to the noises introduced by cloning procedure. For two identical clones the magnitude of fidelity is $F = 2/3$.

Let us now examine the optimal cloning procedure by Eqs. (17), (18) for the BEC state (14) of the EIT-medium.

In the BEC state (14) under the EIT condition for the mean values $\langle X_{f,\xi}^{out} \rangle$ and for the variances $V_{X,f}^{out}$ of output polariton parameters we obtain:

$$\langle Q_{f,\xi}^{out} \rangle = \langle Q_f^{in} \rangle \mp \sqrt{2N} |\alpha_1| |\alpha_2| \cos \varphi, \quad (23)$$

$$\langle P_{f,\xi}^{out} \rangle = \langle P_f^{in} \rangle \mp \sqrt{2N} |\alpha_1| |\alpha_2| \sin \varphi, \quad (24)$$

$$V_{Q,f}^{out} = V_{Q,\xi}^{out} = V_{Q,f}^{in} + 1 - 2 |\alpha_1|^2 |\alpha_2|^2 \cos^2 \varphi, \quad (25)$$

$$V_{P,f}^{out} = V_{P,\xi}^{out} = V_{P,f}^{in} + 1 - 2 |\alpha_1|^2 |\alpha_2|^2 \sin^2 \varphi. \quad (26)$$

The expressions (23)–(26) demonstrate the deviations from “usual” optimal cloning procedure – cf. with Eqs. (19),(20). They are extremal (maximal) for normalized atomic populations $|\alpha_1|^2 = |\alpha_2|^2 = 1/2$ and relative phase $\varphi = \pi/4$. Formally, using an expressions (17), (18) for relevant correlation coefficients (5)–(7) and fidelity parameter F defined in Eq. (22) one can obtain extremal values $C_{1,2}^{(Q)} = C_{1,2}^{(P)} = 4/7$, $C_3^{(Q)} = C_3^{(P)} = 25/49$, $F = 0.8$. However for good quantum cloning procedure we should require the similarity for output and input quadrature components for the scheme in Fig.4 which is possible exactly in adiabatic approximation (3). In particular, in the strong limit when $|\alpha_1| \rightarrow 0$ and $|\alpha_2| \simeq 1$ respectively, we obtain the same results as in Exp. (21) with fidelity $F = 2/3$.

In the paper we are not consider the problem of decoherence of atomic and/or optical system. Although ultracold three level condensate medium under the EIT condition is robust in respect of absorption and Doppler broadening in single pass regime (see e.g. Ref. 4) the decoherence and losses become important especially for long lived quantum memory purposes. Recently various multi-pass protocols have been proposed for retrieval of quantum optical state, verification of quantum storage state and for increasing the fidelity, respectively^{1,21}. Indeed the retrieval procedure for continuous variables consists of mapping of measured light quadrature back into the atomic quadrature variable of the medium with certain feedback gain. In some sense such a technique is similar to the methods proposed early to achieving good QND-measurement of light quadrature component or photon number in signal field

– see e.g. Ref. 11. The problem of retrieval procedure for the schemes proposed by us in Fig.2 and Fig.4 require detail analysis what is not subject of consideration in the paper. However it is important to note that multi-pass protocols obviously are limited by the lifetime of atomic system what in our case of condensate medium is large enough (up to few seconds – see e.g. Refs. 4,5,15).

In Appendix we pay our attention to the problem of stimulated Raman photoassociation process as an real way for possible experimental observation of the phenomena under consideration (in Figs.3,4).

4. Conclusion

In the present paper an opportunity to store quantum state of optical field into the bright and dark polaritons arising in the EIT-like medium is demonstrated. We propose two different schemes for realization of quantum memory for photons. One possibility is connected with the QND recording of two Hermitian quadratures of signal optical field. The relevant correlation coefficients demonstrate possibility to achieve quantum level of information storage in the BEC medium. Another possibility is represented by quantum cloning procedure for signal field in original cloning device offered by us. In last case presence of the EIT effect in condensate medium is absolutely necessary for the optimal cloning procedure under the adiabatic approximation condition.

The physical schemes proposed by us can be useful in problem of quantum cryptography for the first time. Recently, various quantum key distribution (QKD) protocols have been proposed for continuous variables^{9,10,22,23}. In particular, it have been shown (see e.g. Ref. 9) that for individual eavesdropping by beam splitter or cloning machines (when we have less than 3 dB total losses in the channel) these protocols are secure whenever $I_{AB} > I_{AE}$ ($I_{AB(E)}$ is information rate between Alice and Bob (Eve)) with the usage of special reconciliation and privacy amplification procedure. However, in some cases it is possible to create secure key beyond the 3 dB limit when $I_{AB} < I_{AE}$ (or $I_{AB} < I_{BE}$) with the help of appropriate postselection procedure^{22,23}. In general the information rates for Gaussian channels depends on signal-to-noise ratio and, more precisely, on the variance of quadrature components measured by Eve. From our point of view the schemes proposed by us in Fig.2 and Fig.4 can be used by Eve for individual attacks on the communication channel. In this case Eve, for example, can explore simultaneous spin squeezing of the "quadratures" Q_{ξ}^{out} and P_{ξ}^{out} (see Eqs. (25), (26)) when she perform measurements with her clone. The detailed consideration of this problem as well as security of QKD problem in this case is subject of separate analysis.

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Appendix A. Stimulated Raman two-color photoassociation under the EIT condition

Here we analyze the process of stimulated Raman two-photon photoassociation in the presence of the EIT effect. The approach to the problem is based on the stimulated Raman adiabatic passage (STIRAP) procedure resulting in coherent coupling of the long life time levels in free atoms and as a result, in establishment of molecular states. The process can be described in formalism of the Λ -scheme in Fig.1.

In fact, initially we have a free pair of the BEC atoms in the level $|1\rangle$, determined by energy $2E_1$. Then, control field (frequency ω_1) creates the molecules in the excited molecular state $|3\rangle$, i.e. described by energy E_3 . Another laser field (frequency ω_2) removes adiabatically the molecules to the lower state $|2\rangle$ of molecular condensate. Thus, the STIRAP procedure is very closed physically to the EIT observation in three level atomic BEC (cf. Ref. 4 with Refs. 14,24).

To describe the process let us suppose that the signal field is a quantum field characterizing by annihilation photon operator f . In the three coupled condensate modes approximation the Hamiltonian for stimulated Raman photoassociation can be represented in the form¹³:

$$H = \sum_{i=1,2,3} E_i a_i^\dagger a_i + \frac{\hbar}{2} \sum_{ij=1,2,3} \lambda_{ij} a_i^\dagger a_j^\dagger a_i a_j - \frac{\hbar}{2} (\kappa e^{-i\omega_1 t} a_3^\dagger a_1^2 + \Omega_2 e^{-i\omega_2 t} a_3^\dagger a_2 f + H.c.) \quad (A.1)$$

where a_j is the j -th annihilation operator for atoms ($j = 1$), molecules ($j = 2$) and excited molecules ($j = 3$), respectively; the coefficients λ_{ij} characterize the atom-atom, atom-molecule and molecule-molecule scattering in the Born approximation; the parameters κ and Ω_2 specify the Rabi frequencies. Then, in rotating frame (realized by substitution $a_j \rightarrow a_j \exp(-(E_j + \Delta_j)t/\hbar)$) the mean-field equations have the form:

$$\frac{d\langle a_1 \rangle}{dt} = -\gamma_1 \langle a_1 \rangle + i\delta_1 \langle a_1 \rangle + i\Omega_1^* \langle a_3 \rangle, \quad (A.2)$$

$$\frac{d\langle a_2 \rangle}{dt} = -\gamma_2 \langle a_2 \rangle + i\delta_2 \langle a_2 \rangle + i\frac{\Omega_2}{2} \langle f^+ \rangle \langle a_3 \rangle, \quad (A.3)$$

$$\frac{d\langle a_3 \rangle}{dt} = -\gamma_3 \langle a_3 \rangle + i\frac{1}{2}\Omega_1 \langle a_1 \rangle + i\Omega_2 \langle f \rangle \langle a_2 \rangle, \quad (\text{A.4})$$

$$\frac{d\langle f \rangle}{dt} = i\frac{\Omega_2}{2} \langle a_2^+ \rangle \langle a_3 \rangle, \quad (\text{A.5})$$

where $\delta_j = \Delta_j - \sum_{k=1}^3 \lambda_{jk} \langle a_k^+ a_k \rangle$ is effective frequency shift, $2\Delta_1 = (E_3 - 2E_1)/\hbar - \omega_1$, $\Delta_2 = (E_3 - E_2)/\hbar - \omega_2$ are detunings in rotating frame; γ_j defines spontaneous decay rate from j th level. Neglecting by nonlinear character of atom-molecular interaction for free-bound transition we introduce the effective Rabi frequency $\Omega_1 \simeq \kappa \langle a_1 \rangle$.

So, set of Eqs. (A.2)–(A.4) characterize the usual STIRAP procedure for two-color photoassociation effect^{13,14}. Last Eq. (A.5) takes into account the properties of signal field under the STIRAP procedure realization that absolutely necessary in the EIT scheme.

Alternatively, set of Eqs. (A.2)–(A.5) can also be derived directly from the interaction Hamiltonian :

$$H = -\hbar\delta_1 a_1^+ a_1 - \hbar\delta_2 a_2^+ a_2 - \frac{\hbar}{2} (\Omega_1 a_3^+ a_1 + \Omega_1^* a_3^+ a_1) - \frac{\hbar\Omega_2}{2} (a_3^+ a_2 f + f^+ a_2^+ a_3), \quad (\text{A.6})$$

that obviously can be associated with the EIT problem in Λ -systems^{3,5}. Therefore, we can now express the oblique angle θ in the terms of molecular Rabi frequencies according to expression (2).

Thus, the bright and dark polaritons are specified in this case by collective phenomenon of the atom-molecular and optical field excitation. The experimental realization of such a stimulated Raman two color photoassociation is difficult, in principle, due to several physical reasons (see e.g. Ref. 24). But demonstration of dark state resonance performed recently in Ref. 14 is an important step toward the EIT-effect observation.

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